Recent and Potential Advances of the HGPT Methodology

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Abstract. In this paper two recent advances and a potential one, all based on the Heuristic GPT (HGPT) methodology, are described. The first two advances concern, respectively, a method for the on-line monitoring of a subcritical (ADS) system and a method for detecting potential hot spots in a power reactor via prompt response Self Powered Neutron Detectors (SPND). The third one concerns the potential implementation of GPT methods in Monte Carlo codes**.**

Key Words: GPT, hot spot, SPND, Monte Carlo

1. Introduction

Basing on a concept of importance conservation defined first in the field of radiation particles by Kadomtzev in 1957 [1], a heuristically based perturbation theory method was first proposed in 1963 by Usachev for studies of reaction rate ratios [2]. The method was successively extended [3,4,5,6] to include a broader range of functionals in the linear and non-linear domain. In the following, we shall call this method HGPT (Heuristic GPT) to distinguish it from other forms of derivation, in particular, those based on variational and formal derivative techniques [7,8,9], generally known as GPT methods.

In this paper two recent advances and a potential one, all based on the HGPT methodology, are described. The first two advances consist, respectively, in a method for the on-line monitoring of the subcriticality in an ADS reactor and in a method for detecting potential hot spots in a power reactor via flux monitoring by prompt response Self Powered Neutron Detectors (SPND). The third one consists in the potential implementation of the GPT methods in Monte Carlo codes.

2. Subcriticality Monitoring Method

A problem connected with the operation of subcritical (ADS) reactors is posed by the ability of evaluating with sufficient precision their subcriticality level. We illustrate here a general approach to this problem, making use of a derivation of the zero kinetics equations relevant to these systems [10, 11]. These equations are obtained starting from those governing the neutron and the precursor densities and result, in terms of the normalized power P and of the "effective" precursor density ξ_i,

$$l_{\text{eff}} \frac{dP}{dt} = (\rho_{\text{gen}} - \alpha\beta_{\text{eff}})P + \alpha\sum_{i=1}^{I}\lambda_i\xi_i + \zeta(1-P) + \rho_{\text{source}} \qquad (2.1)$$

$$\frac{d\xi_i}{dt} = \beta_{i,eff} P - \lambda_i \xi_i \ , \tag{2.2}$$

where:

$$P(t) = \frac{W(t)}{W_o(1+q)} \quad \left(q = \frac{< \delta \mathbf{\Sigma}_f^T, \boldsymbol{\phi}_o >}{< \mathbf{\Sigma}_{f,o}^T, \boldsymbol{\phi}_o >} \right) \quad \text{(normalized power)} \tag{2.3}$$

$$\xi_i = \frac{< m_{s,o}^* m_i >}{< \mathbf{n}_{s,o}^*, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \ , \quad \text{(i'th effective precursor density)} \tag{2.4}$$

$\bar{\chi} S_{f,o} \boldsymbol{\phi}_o$ being the fission source,

$$l_{eff} = \frac{< \mathbf{n}_{s,o}^*, V^{-1} \boldsymbol{\phi}_o >}{< \mathbf{n}_{s,o}^*, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \quad \text{(effective prompt neutron lifetime)} \tag{2.5}$$

$$\rho_{gen} = \frac{< \mathbf{n}_{s,o}^*, \delta B \boldsymbol{\phi}_o > + \frac{\gamma}{W_o} < \delta \mathbf{\Sigma}_f, \boldsymbol{\phi}_o >}{< \mathbf{n}_{s,o}^*, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \ , \quad \text{(generalized reactivity)} \tag{2.6}$$

with $\delta B = \delta A + \bar{\chi} \delta S_f$, $\delta \mathbf{A}$ accounting for perturbations of absorption, leakage and scattering terms,

$$\rho_{source} = \frac{< \mathbf{n}_{s,o}^*, \delta \mathbf{s}_n >}{< \mathbf{n}_{s,o}^*, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \quad \text{(source reactivity)} \tag{2.7}$$

$$\alpha = \frac{< \mathbf{n}_{s,o}^*, \chi_D S_{f,o} \boldsymbol{\phi}_o >}{< \mathbf{n}_{s,o}^*, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \tag{2.8}$$

$$\bar{\chi} \equiv (1-\beta)\chi_P + \beta \chi_D \tag{2.9}$$

$$\beta_{i,eff} = \frac{\sum_{g=1}^{G} \sum_{j=1}^{J} < n_{s,o,g}^* c_j \chi_{D,g}^j \beta_{i,g}^j \nu \sigma_{f,g}^j \phi_g >}{\sum_{g=1}^{G} \sum_{j=1}^{J} \sum_{i=1}^{I} < n_{s,o,g}^* c_j \chi_{D,g}^j \nu \sigma_{f,g}^j \phi_g >}, \quad \beta_{eff} = \sum_{i=1}^{I} \beta_{i,eff} \tag{2.10}$$

$$\zeta = \frac{1}{< \mathbf{n}^{*}_{s,o}, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \equiv \frac{1 - K_{sub}}{K_{sub}} \quad , \tag{2.11}$$

where

$$K_{sub} = \frac{< \mathbf{n}^{*}_{s,o}, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >}{1 + < \mathbf{n}^{*}_{s,o}, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} , \tag{2.12}$$

with the (power-related) importance $\mathbf{n}^{*}_{s,o}$ governed by the equation

$$B^{*}_{o} \mathbf{n}^{*}_{s,o} + \frac{\gamma}{W_o} \boldsymbol{\Sigma}_{f,o} = 0 \tag{2.13}$$

Coefficient K_{sub} merges into K_{eff} (the multiplication coefficient relevant to the fundamental eigenfunction) with the system approaching criticality.

At unperturbed, steady state conditions $P=P_o=1$ and $\xi_i = \beta_{i,\,eff}/\lambda_i$.

In the following, a method is described [11] for determining experimentally the subcriticality level basing on the above concepts. It shall be referred to as Power Control based Subcriticality Monitoring (PCSM) method.

2.1. The PCSM method

Consider a change of a (calibrated) control rod position. This would correspond to an experimental reactivity value $\left(\delta k_{eff} / k_{eff}\right)^{\mathbf{exp}}_{B}$. The associated value $\rho^{exp}_{gen,B}$ of the generalized reactivity could be assumed as

$$\rho^{\mathbf{exp}}_{gen,B} = \rho^{cal}_{gen,B} \, f_b \quad , \tag{2.14}$$

with $\rho^{cal}_{gen,B}$ given by Eq. (2.6) and f_b a bias factor given by the expression

$$f_b = \frac{\left(\delta k_{eff} / k_{eff}\right)^{\mathbf{exp}}_{B}}{\left(\delta k_{eff} / k_{eff}\right)^{calc}_{B}} \quad , \tag{2.15}$$

with $\left(\delta K_{eff} / K_{eff}\right)^{\mathbf{exp}}_{B}$ obtained by a standard control rod calibration and $\left(\delta K_{eff} / K_{eff}\right)^{calc}_{B}$ given by the expression

$$\left(\frac{\delta K_{eff}}{K_{eff}} \right)^{calc}_{B} = \frac{< \boldsymbol{\phi}^{*}_{o}, \; \delta B_B \boldsymbol{\phi}_o >}{< \boldsymbol{\phi}^{*}_{o}, k^{-1}_{eff} \, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \quad , \tag{2.16}$$

$\boldsymbol{\phi}^{*}_{o}$ being the standard adjoint flux and δB_B the perturbation of the (diffusion, or transport) operator relevant to the control rod insertion.

Likewise, the source reactivity ρ_{source}^{exp}, associated with a given measured source change $\delta \mathbf{s}_n^{exp}$, could be assumed as

$$\rho_{source}^{exp} = \frac{< \mathbf{n}_{s,o}^{*}, \delta \mathbf{s}_n^{exp} >}{< \mathbf{n}_{s,o}^{*}, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \quad . \tag{2.17}$$

Recalling the definition of importance and assuming that the perturbation of the source corresponds to a (measured) fractional change of its strength, represented by $\delta s_n^{exp}/s_n$, we obtain the expression

$$\rho_{source}^{\mathbf{exp}} = \frac{\delta s_n^{\mathbf{exp}}}{s_n} \frac{1}{< \mathbf{n}_{s,o}^{*}, \bar{\chi} S_{f,o} \boldsymbol{\phi}_o >} \equiv \frac{\delta s_n^{\mathbf{exp}}}{s_n} \frac{1 - K_{sub}}{K_{sub}} \quad . \tag{2.18}$$

If we consider changes of the control rod and of the external source, such that the power level remains unaffected, we may write, considering Eqs.(2.1) and (2.2) at steady state conditions,

$$\rho_{gen,B}^{exp} + \rho_{source}^{exp} = 0 \ . \tag{2.19}$$

Substituting expression (2.18), we obtain finally the expression

$$K_{sub} = \frac{\delta s_n^{\mathbf{exp}} / s_n}{\delta s_n^{\mathbf{exp}} / s_n - \rho_{gen,B}^{\mathbf{exp}}} \ . \tag{2.20}$$

To note that $\Delta s_n^{\mathbf{exp}} / s_n$ and $\rho_{geB}^{\mathbf{exp}}$ have opposite signs.

So, by properly adjusting the external source strength for compensating a control rod insertion, the subcriticality index (1-K_{sub} / K_{sub} can be estimated. The adjustments should be effected gradually at steps, so to keep the overall power practically unaltered.

2.2. Conclusions

The PCSM method is proposed for safely determining the subcriticality level of an ADS system without significantly interfering with its normal operation. The method consists in:

- a precalibration of a control rod. The dedicated control rod should be of limited worth so that in any circumstance the system maintains well below criticality conditions. A relationship between a control rod position change and the corresponding reactivity alteration may then be established;

- during operation, a small, slow insertion of the control rod should be associated with an adjustment of the accelerator current, so that the count rate of a neutron detector in an out-of-core position is maintained constant, so that the same power level is maintained;

- determining the value of K_{sub}, making use of Eq. (2.20).

A numerical simulation exercise considered in view of an experiment on a TRIGA reactor at subcritical conditions demonstrates the potentiality of the proposed methodology [12].

3. Hot spot identification method by sensitivity analysis and probabilistic inference

Through the use of the Generalized Perturbation Theory (GPT) techniques and of those of probabilistic inference [13] a method has been developed [14] for the detection of possible hot spots during the operation of a nuclear reactor on the basis of on-line measurements of the neutron flux. These measurements are assumed to be made by making use of Self Powered Neutron Detectors (SPND), also named 'collectrons' [15].

The method has first been conceived for its use in thermal reactors, in particular in PWRs, but it could be as well extended to fast reactors if efficient SPND detection techniques are developed for these systems [16]. The method is based on the calculation of the sensitivity coefficients of integral quantities measured by the collectrons in relation to parameters representative of the hot spot, and on a consistent use of probabilistic inference techniques.

The methodology takes into account the errors associated with the measurements. It also allows to evaluate the effect on the quality of the detections as a result of possible failures of the measuring instruments during the core life. Such evaluation can be useful for defining an adequate protection strategy in terms of quality, number and distribution of collectrons.

On this subject a simulation exercise has been made [17]. The results obtained have confirmed the validity of the proposed method.

3.1. Method

Let us assume that a fixed number (N) of collectrons are positioned in the core of a given reactor. Let us assume then a number (M) of hypothetical hot spot positions[1]. In order to simplify the presentation of the method, we assume that the core may be represented in two-dimension (x, y) geometry. We assume also the hypothesis that in each position m a constant value is maintained of the ratio r_m between the maximum and the average linear power densities p_m^{max} and $\bar{p}_m$, i.e.:

$$r_m \equiv p_m^{max} / \bar{p}_m \quad . \tag{3.1}$$

An alteration $\Delta\bar{p}_{m'}$ at position m' due to a flux surge by an hypothetical hot spot may be written

$$\Delta\bar{p}_{m'} = \gamma \frac{\left\langle \Delta\boldsymbol{\phi}^T \boldsymbol{\Sigma}_f \right\rangle_{m'}}{L_c} \quad , \tag{3.2}$$

γ being the number of energy units per fission, L_c the core height and $\left\langle \Delta\boldsymbol{\phi}^T \boldsymbol{\Sigma}_f \right\rangle_{m'}$ the fission rate change at position m'. It is this latter quantity that the proposed method intends to evaluate.

Let us then denote by $p_m^{max,1}$ a first threshold for the linear power density value relevant to each of the M possible hot spots considered, above which an attention warning would be triggered, and a second threshold $p_m^{max,2}$ above which a plant shutdown would take place.

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[1]The power increase in a hot spot within a given fuel pin is associated with a corresponding increase of the fission neutron source in the pin itself. This fission source increase is here considered spread into the whole fuel assembly, which would then define the volume associated with the hot spot position itself.

From the analysis of the measurements of quantities Q_n given by the collectrons, the possibility of the presence, or not, of an hot spot condition in one, or more, of the M hypothetical positions considered has to be evaluated in relation to the assigned thresholds.

3.2. Theory

The quantities considered by the methodology are the ratios between the capture rates of the detectors within the (N) collectrons and the fission rate integrated over the whole core, i.e., in relation to detector n,

$$R_n \equiv \frac{Q_n}{F_{core}} = \frac{< \boldsymbol{\Sigma}_r^T \boldsymbol{\Phi} >_{(n)}}{< \boldsymbol{\Sigma}_f^T \boldsymbol{\Phi} >_{(core)}} \ , \tag{3.3}$$

where $\boldsymbol{\Sigma}_r$ is the vector of the capture cross sections of the collectron detector.

A neutron source ('external', that is, not associated with fission reactions in the fuel) is then considered in correspondence to each of the M fuel element positions, given by the expression

$$s_m \boldsymbol{\chi} \xi_m(\mathbf{r}) \ , \tag{3.4}$$

where s_m is a scalar quantity, $\boldsymbol{\chi}$ is a vector representing the fission spectrum and $\xi_m(\mathbf{r})$ is a function equal to one inside the volume associated with the m'th potential hot point position and equal to zero outside. An external source with a fission spectrum, in relation to its effect on the collectrons, may very well simulate the fission reaction rate burst produced by an hot spot event. At this point we introduce the sensitivity coefficients

$$w_{n,m} = \frac{dR_n}{ds_m} \ . \tag{3.5}$$

According to the generalized perturbation theory (GPT) [5] these coefficients can be calculated using the expression

$$w_{n,m} = < \boldsymbol{\Psi}_n^{*T} \boldsymbol{\chi} \xi(\mathbf{r}_m) > \ , \tag{3.6}$$

where $\boldsymbol{\Psi}_n^*$ is the importance function associated with functional R_n and obeying equation

$$\mathbf{B}^* \boldsymbol{\Psi}_n^* + \left(\frac{\boldsymbol{\Sigma}_r}{Q_n} \xi_n(\mathbf{r}) - \frac{\boldsymbol{\Sigma}_f(\mathbf{r})}{F_{core}} \right) = 0 \ , \tag{3.7}$$

B being the operator governing the neutron flux and $\xi_n(\mathbf{r})$ a function equal to one inside the volume of the detector material of the n'th collectron (or an equivalent volume in which it might have been homogenized) and zero outside.

Each sensitivity coefficient $w_{n,m}$ defined by Eq. (3.6) represents the contribution of a fission neutron from position m to the measurement in collectron n, as represented by the response R_n defined by Eq. (3.3). These coefficients form then vector

$\mathbf{w}_m \equiv \left| w_{1,m} \quad \cdots \quad w_{N,m} \right|^T$ characteristic of each of the M possible hot spot positions. In a way, it may be considered the 'signature' of each position.

Given a series of neutron flux detections R_n^{ex}, to which a corresponding vector $\mathbf{R}^{ex} \equiv \left| R_1^{ex} \quad ... \quad R_N^{ex} \right|^T$ may be associated, a potential hot spot search will start once the values of one or more components R_n^{ex} depart significantly (i.e., beyond given uncertainty margins) from the nominal values $R_{o,n}$. In this case the candidate positions will be chosen among those for which the normalized distribution of components of vector $\mathbf{w}_m$ will be closer to the normalized distribution of components of vector $(\mathbf{R}^{ex} - \mathbf{R}_o)$, where $\mathbf{R}_o \equiv \left| R_{o,1} \quad ... \quad R_{o,N} \right|^T$, i.e., among those (say, M') for which, within a fixed range of uncertainty based on measurement accuracies, is minimal the sum

$$S_m = \sum_{n=1}^{N} \left[\alpha_1 (R_n^{ex} - R_{o,n}) - \alpha_{2,m} w_{n,m} \right]^2 \quad , \tag{3.8}$$

where α_1 and $\alpha_{2,m}$ are normalization coefficients, i.e.,

$$\alpha_1 = \frac{1}{\sum_{n=1}^{N} (R_n^{ex} - R_{o,n})} \quad , \qquad \alpha_{2,m} = \frac{1}{\sum_{n=1}^{N} w_{n,m}} \quad . \tag{3.9}$$

A criterion to be used for determining the set of the M' candidate positions could be that of: first, identifying the position $\overline{m}$ such that $S_{\overline{m}}$ corresponds to the minimum sum and, secondly, selecting those positions for which the sum

$$S_m \le (S_{\overline{m}} + \varepsilon_{S\overline{m}}) \tag{3.10}$$

where $\varepsilon_{S\overline{m}}$ is the standard deviation (or a multiple of it, to be more conservative) of the sum $S_{\overline{m}}$.

A numerical simulation has been made [17] relevant to a simplified, medium size PWR system [18]. For the demonstration purposes of this study a bidimensional XY geometry has been considered. The main simplification made with respect to the original project is the absence of control rods. The ERANOS code [19] has been used for the analysis. The calculations were made in diffusion approximation using a 15 group cross-section library. As regards the detector material in the collectrons, Co^{59} has been chosen. The presence of this detector has been simulated by replacing in a fuel element 0.0436% vol of water with cobalt. The simplifications taken are justified by the fact that in the exercises considered we are merely interested in testing the methodology. The validity of the results obtained should however be reasonably extrapolable to more realistic configurations.

In Fig. 1 the positions of the elements containing the collectron devices are indicated together with the positions of the fuel elements in which the occurrence of a potential hot spot are considered (limited, for simplicity, to the first quadrant).

For this simulation exercise, the 'detections' Q_n^{ex} have been assumed corresponding to a set of quantities randomly sorted according to a Gaussian distribution law characterized by average values Q_n^{cal} and a 5% standard deviation. Adopting the method illustrated above, it is straightforward to identify the hot spot candidates, i.e., the positions in correspondence to which the sum defined by equation (3.8), within a given uncertainty range, is minimal. For illustration, in Table 1 the values of such sum are given for each fuel element position. It may be seen that the minimum value corresponds to position n° 8, as expected. Once the position of a possible hot spot candidates is identified, and the sensitivity coefficients related to it are determined, we may use the probabilistic inference methods for estimating the value of the

neutron source, here viewed as a simulation of the fission neutron source increase produced by the hot spot occurrence, and the statistical error associated with it.
In Table 2 the results are shown relevant to the exercise in which the hot spot event (simulated by an increase of the fission source rate in a given fuel element equal to 1) is estimated by probabilistic inference techniques, considering various degrees of collectron system degradation.

3.3. Conclusions

The results obtained with the simulation exercise indicate how the methodology proposed may be used with success as a hot spot identification tool by fully exploiting the information available from a collectron detection system implemented in a nuclear reactor plant.

The hot spot detection method described above may be useful also at a reactor design stage. An extensive analysis relevant to the spatial distribution of the collectrons and to their possible failure sequences could allow, in fact, to identify optimal configurations based on plant engineering and economic considerations.

The method proposed might be considered for detecting as well, beside anomalous flux surges, 'hot spots' produced by channel flow blockages producing a local temperature increase, in turn causing a reduction of the fission reaction rate due to the augmented absorption of the fertile material.

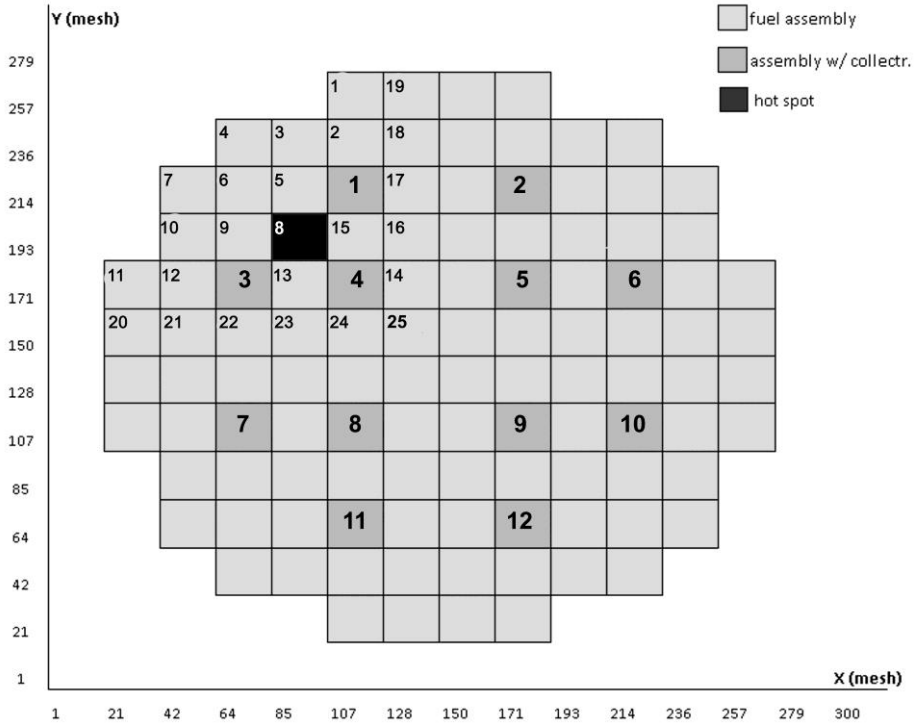

Fig. 1 . Core layout with the positions of collectrons and hot spot

Table 1. Values of the sums defined by Eq. (3.8)

Hot spot position	SUM
1,2,…,6,7,9,10,…,24,25	>6.00E+00
8	**4.63E-03**

Table 2. Results from simulation (Hot spot at position 8)

(collectron detections assumed independent, each with 3% error)

Degradation (Failed collectrons)	Hot spot candidates positions (satisfying Eq.3.10)	Hot spot		
		Simulated	Estimated	Stand. Dev.
0	8	1.000	1.003	0.079
1	8	1.000	1.003	0.094
1,2	8	1.000	1.004	0.095
1,2,3	8	1.000	1.008	0.123
1,2,3,4	8	1.000	1.005	0.153
1,2,3,4,5	8	1.000	1.007	0.162
1,2,3,4,5,6,7	6,**8**,9,15	1.000	1.006	0.176 (min)

4. Use of the EGPT Methodology with Monte Carlo

A method under development [20] considers the use of the Monte Carlo methodology for GPT analysis, in particular relevant to ratios of functionals bilinear with the real and adjoint fluxes (as the reactivity worths). The basic idea is the adoption of the Equivalent Generalized Perturbation Theory (EGPT) [21] modality of GPT, which transforms the problem of solving inhomogeneous equations into that of solving homogeneous ones with the governing operators properly modified according with the functional (response) under investigation. The method is based on the existing capability of the MCNP6 code [22] by which it is possible to estimate the adjoint function by using the iterative mechanism of the KCODE modules [23] and by weighting a dedicated tally to obtain estimates of reactivity changes.

Consider a reactivity worth

$$\rho_c = \frac{<\phi^* \delta_c B \mathbf{n}_{(c)}>}{<\phi^* F_{(c)} \mathbf{n}_{(c)}>} \equiv \frac{<\phi^* \delta_c B_{(c)} \mathbf{n}_{(c)}>}{<\phi^* F \mathbf{n}_{(c)}>} \quad , \qquad (4.1)$$

where B ($\equiv A+\lambda F$) is the governing operator, $\delta_\chi B = \delta_c A + \lambda \delta_c F$ and $\delta_c B_{(c)} = \delta_c A + \delta_c \lambda \delta_c F \equiv \delta_c A - \rho_c \delta F$. Quantities $\phi_{(c)}$, $B_{(c)}$ and $F_{(c)}$ are relevant to the system state altered by the change $\delta_c B$ of B.

Along with the GPT methodology, the following perturbation expression may be written, after an alteration of system parameters implying a change $\delta_s B$ of the governing operator,

$$(\delta_s \rho_c)_{GPT} = \frac{<\psi^*_{c(c)} \delta_s B_{(c)} \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} + \frac{<\phi^* \delta_s B \psi_c>}{<\phi^* F \phi_{(c)}>} - \rho_c \frac{<\phi^* \delta_s F \phi_{(c)}>}{<\phi^* F_{(c)} \phi_{(c)}>} \qquad (4.2)$$

where $\psi^*_{c(c)}$ and ψ_c are the importance functions associated to the functional defined by Eq. (4.1).

Along with the EGPT methodology [21], Eq. (4.2) results equivalent to:

$$(\delta_s\rho_c)_{EGPT} = \frac{<\phi^*_{(c)}\delta_s B_{(c)}\phi_{(c)}>}{<\phi^*_{(c)}F_{(c)}\phi_{(c)}>} - \frac{<\phi^*\delta_s B\phi>}{<\phi^* F\phi>} \quad . \tag{4.3}$$

The r.h.s, of this equation could be interpreted as the difference of the first-order reactivity changes induced by $\delta_s B$ at perturbed (by $\delta_c B$) and unperturbed conditions.

Preliminary calculation tests [20] relevant to a simplified model of a HTGR reactor demonstrate the potentiality of the proposed methodology for GPT analysis with Monte Carlo of functionals bilinear with the real and adjoint neutron fluxes, as the reactivity worths.

The extension of the use of this methodology for the analysis of functionals of only the neutron flux, as the reaction rate ratios, is also envisaged.

To be reminded also the implementation of perturbation techniques into the SERPENT code [24]. This code allows in effect to calculate sensitivity coefficients of a given quantity (response) with respect to the system parameters. However, the method it uses requires a Monte Carlo calculation run for each parameter considered, whereas, with the method proposed above, only simple, fast integration (sum) operations would be necessary for any number of parameters, on the basis of pre-calculated quantities at unperturbed conditions. As regards the computation time, this fact may become crucial when large systems are considered for analysis.

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