Analysis of Various Approximations in Neutronic Calculations of Transient in Fast Reactors

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Abstract. The paper considers the kinetic calculations of several tests associated with movement of control rods in fast reactors. Each test has been calculated by a direct numerical solution of the transient neutron transport equation based on a diffusion theory and by different approximate schemes of solution of the original equation. The calculation results demonstrate that some approximations that are successfully used for the calculation of similar problems in the thermal reactor are not able to provide acceptable solution accuracy for the fast reactor. Results of the analyses of the different solution schemes are presented. It is found that solutions obtained using combined schemes based on an improved quasi-static approximation are preferred.

Key Words: transient neutron transport equation, approximate schemes of solution, kinetic calculation, error of solution

1. Solution Methods for the Time-Dependent Diffusion Equation

A reliable analysis of dynamic processes during normal operation, design-basis and beyond the design-basis accident is required for safety substantiation of nuclear reactors. A module of solution of transient neutron transport problem is an essential part of such complex codes.

The neutron transport problem can be solved numerically without the use of the approximate forms of solution. Parameters that actually observed in the reactor (such as the neutron flux density depending on the time and space, the concentration of precursors of delayed neutrons and the delayed neutron fraction at each division of an original nuclide) are used in solving of the direct problem [1, 2, 3]. Numerical solution of the direct problem provides high result accuracy and qualitatively describes various processes occurring in a reactor. A significant drawback of the direct numerical solution is the significant computation time so it is more often used to solve important practical problems or verification of the results of the approximate solution [4].

In practice, approximate methods are widespread for solving spatial time-depended tasks. The most common ones are the equation of point kinetics, adiabatic approximation, quasi-static approximation and the improved quasi-static approximation. For definiteness in the paper the multi-group diffusion approximation is considered. In approximate methods of solution a neutron flux $\phi(\mathbf{r}, E, t)$ is factorized into a product of shape function $\varphi(\mathbf{r}, E, t)$ and amplitude function $T(t)$ [4, 5, 6]:

$$\phi(\mathbf{r}, E, t)=T(t)\cdot\varphi(\mathbf{r}, E, t). \quad (1)$$

Symbols: $\mathbf{r}$ — coordinate of the reactor point, E — neutron energy, t — time. It is assumed that the basic time dependence is described by the amplitude function and shape function changes little with time. Substituting the representation (1) to the transient neutron transport

equation the system of equations to determine the amplitude function T(t) and shape function φ(**r**, E, t) are obtained:

$$\begin{cases} \dfrac{\partial T(t)}{\partial t} = \dfrac{(\rho - \beta_{\text{eff}})}{\Lambda} T(t) + \sum\limits_{j=1}^{JD} \sum\limits_{n=1}^{N} \lambda_j^n C_j^n(t) + \dfrac{S(t)}{\Lambda}, \\ \dfrac{\partial C_j^n(t)}{\partial t} = -\lambda_j^n C_j^n(t) + \beta_{j,\text{eff}}^n T(t), \end{cases}$$
$$\frac{1}{\upsilon(\mathrm{E})} \frac{\partial \varphi(\mathbf{r}, \mathrm{E}, t)}{\partial t} + \frac{1}{\upsilon(\mathrm{E})} \varphi(\mathbf{r}, \mathrm{E}, t) \frac{1}{T(t)} \frac{\partial T(t)}{\partial t} = \tag{2}$$
$$= \left[\mathbf{F}_p - \mathbf{M} \right] \varphi(\mathbf{r}, \mathrm{E}, t) + \frac{1}{T(t)} Q_d(\mathbf{r}, \mathrm{E}, t) + \frac{1}{T(t)} Q(\mathbf{r}, \mathrm{E}, t).$$

Symbols: ρ is the reactivity, β_{eff} is the effective fraction of delayed neutrons, Λ is the prompt neutron lifetime, C_j^n is the integral concentration of delayed neutron predecessors, S(t) is the effectiveness of external neutron source, $\beta_{j,\text{eff}}^n$ is the effective fraction of delayed neutrons of j^{th} group of n^{th} nuclide, **F** is the prompt fission operator, **M** is the migration and loss operator, Q_d(**r**, E, t) and Q(**r**, E, t) are the sources of delayed and external neutrons respectively.

Different approximations of the original problem can be obtained using various assumptions and simplification in formulation of the shape function.

If we postulate that the shape function is completely time-independent than the equation for the shape function takes the form:

$$(\mathbf{M}_0 - \lambda \mathbf{F}_0)\, \varphi_0(\mathbf{r}, \mathrm{E}) = 0. \tag{3}$$

This approximation is called the point kinetics equation. Shape function is determined only once at the start of the calculation. Therefore, this approach is used for sufficiently small deformations of the neutron field shape (weak perturbations of reactor environment properties), for example, for description of an asymptotic behavior. Reactivity is determined outside of the solution algorithm, and it can be represented, for example, in a tabular form. The point kinetics equations provide a maximum speed of receipt of the results.

If we neglect the time derivative in equation (2) and combine the source of delayed neutrons with the source of prompt neutrons, so it is possible to obtain an adiabatic approximation:

$$(\mathbf{M} - \lambda \mathbf{F})\, \varphi_k(\mathbf{r}, \mathrm{E}) = 0. \tag{4}$$

The adiabatic approximation can be used to describe spatial time-depended behavior of the reactor for a sufficiently slow change of power. The method provides a good speed of receipt of the results.

If we neglect only the time derivative in equation (2) and the concentration of delayed neutron precursors is determined by the equation

$$c_j(\mathbf{r}, t) = \beta_j \int_{-\infty}^{t} dt' e^{-\lambda_j (t - t')} \int dE \nu \Sigma_f(\mathbf{r}, E) \phi(\mathbf{r}, E, t) \tag{5}$$

then it is possible to obtain the equation of the quasi-static approximation. This method provides sufficient speed of receipt of the results.

The reactivity is determined through the eigenvalues of the stationary reactor states in the adiabatic and quasi-static approximation. For the kinetic solution the asymptotic state of non-

steady process that would be implemented in the real transient after hundreds of seconds after the disturbance is considered.

In the improved quasi-static approximation the time derivative and the formation of delayed neutrons taking into account history of transient is determined. This approach presupposes the existence of two computational grids: the smaller one to determine the amplitude function and the larger one for shape functions. Considering that the recomputation of the shape function is the most costly process, such a grid partitioning allows to reduce the calculation time.

It is considered that the scheme of the improved quasi-static approximation almost corresponds to the full numerical solution of the original problem. The accuracy of the improved quasi-static solution is mainly related to the reactivity that is determined by the first-order perturbation theory using the importance function obtained before the initial perturbation to reactor was made [7]:

$$\rho_i = \left\langle \varphi_0^+ \delta\Sigma_m \varphi_{i-1} \right\rangle / CND . \qquad (6)$$

Symbols: i — time step, Σ_m — cross-sections, brackets $\langle\ \rangle$ denote integration over all phase space, CND — the value of fission neutrons.

First, during a long transient, reactor can pass through many critical states that are described by different (from initial) distribution of the neutron flux and the importance function. The initial importance function that has a very indirect relationship to the analyzed state at the particular time is used in determining of the reactivity.

Secondly, from the experience of using the formula (6) it follows that reactivity estimated with the first-order perturbation theory and its actual value can vary by 100% or more in the calculation of fast reactors. This variance from the analogous estimates in thermal reactors is due to the significant heterogeneity of perturbations, which is typical for fast reactors (control rods, sodium-void reactivity effect, etc.). For example, the design of the control rods in fast reactors, as a rule, occupies the entire channel or a larger part of it, so the rod has substantially greater heterogeneity, than the control rods in thermal reactors because in a thermal reactor absorber elements are distributed between the fuel rods of the assembly, i.e. the placement of the absorber is substantially more homogeneous. Thus, the algorithm of the improved quasi-static approximation can provide very mediocre result.

Simplified schemes of the solution are limited to the area of their application because of their inherent assumptions. Therefore, a substantiation of possibility of application of the specific approximation schemes is required for calculation of a concrete task. All these simplified solution schemes are characterized by some common features. First of all, the original transient problem is divided into a number of subtasks of determination of reactivity, the amplitude functions, the shape function, etc. Moreover, during the solution it is necessary to operate by some speculative options that, in general, could be normalized arbitrarily, although in practice they are tended to assign a certain physical interpretation. Such parameters like the effective fraction of delayed neutrons, the prompt neutron lifetime, the integral concentration of delayed neutron predecessors, the effectiveness of external neutron source, the value of fission neutrons and so on. The arbitrariness of these parameters is due to the fact that the importance function used to their calculation is not well-defined, since the corresponding critical system is partly arbitrary.

It is particularly worth to emphasize that for the solution of the direct problem without approximations there is no need to determine reactivity. The concept of approximate methods of solution of transient neutron transport equation is that the perturbation of the reactor is understood as the introduction of non-zero values of the reactivity $\rho \neq 0$, i.e., reactivity is

understood as the cause of transient and leads to the subsequent change of power. In fact, a cause of the reactor perturbation is the change of the properties of its environment, which leads to a perturbation of the cross sections with the subsequent redistribution of the neutron flux and, consequently, power.

After considering the strengths and weaknesses of the most popular schemes for the solution of transient neutron transport equation, two methods of combined approximation were formulated. Both methods are based on the classic improved quasi-static approximation and the main difference lies in the method of determining the reactivity.

In the first combined approximation the reactivity is determined by the formula of the General theory of perturbations [7, 8] $\rho = \left\langle \varphi_0^+ \delta\Sigma_m \varphi_k \right\rangle / CND$, based on the value function φ_0^+ of the initial state and shape function φ_k of the final stationary state. This way of reactivity calculation is less sensitive to heterogeneity of perturbations compared with the first-order perturbation theory, but the problem of choosing the distribution of importance function when the reactor passes through other critical states remains relevant.

In the second combined approximation the reactivity is determined by eigenvalues k_{ef} of two conditional critical tasks for the initial and perturbed reactor states without the use of importance function:

$$\rho = \left(1/k_{ef}^{\text{init.}} - 1/k_{ef}^{\text{pertub.}}\right) k_{ef}^{crit} \ .$$

The resulting reactivity is used to determine reactor power with the subsequent calculation of shape function $\varphi^g(\mathbf{r}, t)$. This algorithm is not sensitive to heterogeneity of disturbances and devoid of the difficulties of the selection of the reactor state to calculate the importance function.

In practice, we often have to meet challenges associated with the movement of control rods in the reactor. These tasks can be characterized by a large local disturbance of the environment of the reactor. Point kinetics and adiabatic approximation, as a rule, cannot provide acceptable solution accuracy.

A choice of the optimal algorithm for the solution of transient that carried out according to the results of two test problems is demonstrated below.

2. Task 1

This task demonstrates the accuracy of the circuit solution in modelling of local perturbations in the fast reactor. The calculations were performed in the multigroup diffusion approximation in three-dimensional geometry. The fast reactor core consists of uranium-plutonium nitride fuel surrounded by lead reflector and steel protection. Two steady states of the core were simulated:

1. In state 1 all control rods were completely taken out the core;
2. In state 2 all control rods were fully inserted into the core.

Using different schemes of the approximate solution, it is essential to evaluate the worth of all control rods in the reactor for transition from the state 1 to the state 2 and on the contrary from the state 2 to the state 1, i.e. for the input and output of all control rods respectively.

Reactivity evaluation was carried out in the following ways:

1. From the second combined approximation, i.e. according to eigenvalues of the two steady states;
2. From improved quasi-static approximation, i.e. according the first-order perturbation theory using the shape functions φ and importance function φ^+ of initial state;
3. Similar to item 2, but using the importance function like a constant ($\varphi^+=const$);
4. From the first combined approximation, i.e. according general perturbation theory using the importance function φ^+ of the initial state and the shape function φ of an end state;
5. Similar to item 3, but using a shape function φ of an end state;
6. Similar to item 2, but using a shape function φ and importance function φ^+ of an end state.

Table 1 shows the results of the calculations.

The worth of all control rods in the reactor must be the same in absolute value both in a case of their insertion in the core and their withdrawing from it. At item 1 for both cases of reactivity calculation using the same pair of eigenvalues (k_{ef}^{without} and k_{ef}^{with}) the deviation of reactivity is absent. Calculation at item 4 slightly reacts upon the heterogeneity of local perturbations with error ≈1%. The calculations at items 2 and 6 show significant differences in the worth of the control rods more than 100%. The main contribution to the reactivity determination by the formula of the first-order perturbation theory makes the distribution of shape function because the replacement of the distribution of the importance functions to a constant value (items 3 and 5) almost does not affect the result.

TABLE 1. REACTIVITY EVALUATION FOR THE REACTOR TRANSITION FROM THE STATE 1 TO STATE 2 AND, ON THE CONTRARY, FROM STATE 2 TO STATE 1, %Δ*K*/*K*

Item	Reactivity determination	Transition (1→2)	Transition (2→1)	Error,%
1	$\frac{1/k_{ef}^{\text{init.}} - 1/k_{ef}^{\text{finish}}}{k_{ef}^{crit.}}$	-8.66	8.66	0
2	$\frac{\left\langle \varphi_{\text{init.}}^{+g} \delta\Sigma_m \varphi_{\text{init.}}^{g} \right\rangle}{CND}$	-17.69	4.32	>100
3	$\frac{\left\langle 1 \cdot \delta\Sigma_m \varphi_{\text{init.}}^{g} \right\rangle}{CND}$	-17.88	3.06	>100
4	$\frac{\left\langle \varphi_{\text{init.}}^{+g} \delta\Sigma_m \varphi_{\text{finish}}^{g} \right\rangle}{CND}$	-8.76	8.68	≈1
5	$\frac{\left\langle 1 \cdot \delta\Sigma_m \varphi_{\text{finish}}^{g} \right\rangle}{CND}$	-3.03	17.82	>100
6	$\frac{\left\langle \varphi_{\text{finish}}^{+g} \delta\Sigma_m \varphi_{\text{finish}}^{g} \right\rangle}{CND}$	-4.30	17.62	>100

3. Task 2

This task is intended to reveal the accuracy of different solution schemes during reactor transition through the other critical states. Two stationary reactor states characterized by the same eigenvalues and various positions of control rods were simulated. In the reactor model control rods were located symmetrically forming three rings. The inner ring consists of 6 shim rods. Two critical reactor states were used for solving the task. In the first state, 3 of 6 shim rods of the inner ring (rotating through one) were taken out the core, while the remaining three rods were completely inserted. In the second state those 3 shim rods that had been introduced, were fully extracted from the core, and conversely the remaining three of compensatory rods were introduced. Positions of other control rods have not been changed.

The reactivity of the reactor transitions from the critical state 1 to the critical state 2 and conversely from state 2 to state 1 was evaluated. For reactivity calculation tools and algorithms described in the previous section were used. Table 2 shows the results of the calculations.

The eigenvalues for both critical states are the same, so the both transitions from one critical state to another at item 1 were evaluated in a reactivity of 0% Δk/k. Reactivity calculated at item 4 is small in absolute value. The reactivity values defined at items 2 and 6 significantly differ from zero, and the distribution of importance function (at items 3 and 5) does not affect the result essentially.

TABLE 2. REACTIVITY EVALUATION FOR THE REACTOR TRANSITION FROM THE CRITICAL STATE 1 TO CRITICAL STATE 2 AND FROM STATE 2 TO STATE 1, %Δ*K*/*K*

Item	Transition (1→2)	Transition (2→1)
1	0.00	0.00
2	-1.82	1.82
3	-2.08	2.08
4	-0.014	0.014
5	-2.08	2.08
6	-1.82	1.82

This task was calculated by means of the direct numerical solution of transient neutron transport equation. In each calculation a single instantaneous transition of reactor from one critical state to another followed by tracking of its power were simulated. The values of reactivity are determined by processing the temporal behaviour of power. Figure 1 shows the results of the calculation of the transient before the reactor had achieved an asymptotics.

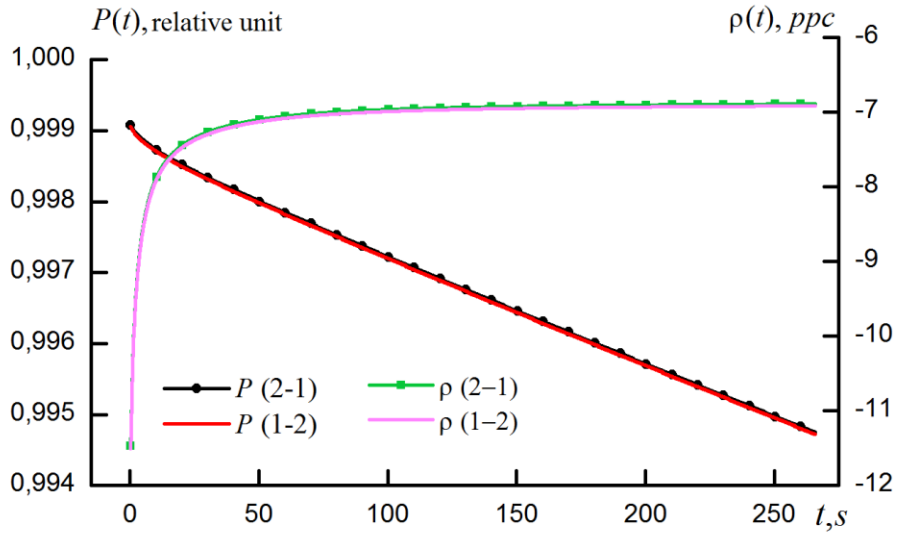

FIG. 1. Graphs of power P(t) and reactivity ρ(t) after a single reactor transition from the critical state 1 state 2 (1→2) and, conversely, of from the critical state 2 to 1 (2→1)

The graphs show the results of these two calculations coincide well with each other. The reactor transition from one critical state to another leads to an abrupt change in the distribution of neutronic characteristics in the six channels with the shim rods of the inner ring. Consequently, spatial distribution of neutron field φ(**r**, E, *t*) and concentration of delayed neutron predecessors $C_j^n(\mathbf{r},t)$ are upset and it is the cause of transient on delayed neutrons. Due to the core symmetry these processes are the same as for the transition from critical state 1 to state 2 and from 2 to 1. Therefore, the reactor power drops in all the calculated steps and respectively reactivity is estimated as negative.

Transient problem in which at each time step reactor transitioned from one critical state to another was calculated. Figure 2 shows the results of this computational experiment.

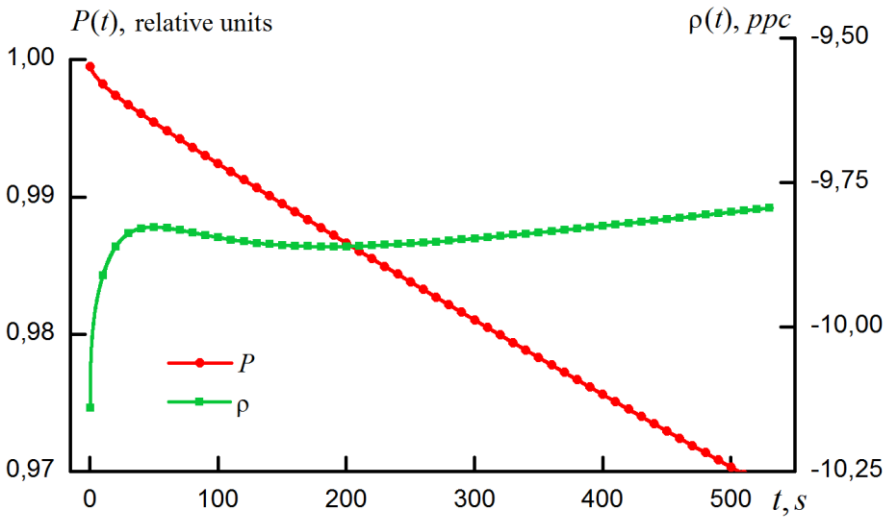

FIG. 2. Graphs of power P(t) and reactivity ρ(t) for the case of multiple instantaneous reactor transitions from one critical state to another

Since the calculated step is significantly less than time of reaching of asymptotics, each reactor transition to another critical state leads to a new "reverse" redistribution of reactor local characteristics. This is the reason for power decreasing, the processing of which yields to negative reactivity.

Returning to the table 2, it is important to emphasize that when using perturbation theory (items 2-6) the transition of the reactor from the critical state 2 to state 1 is evaluated to positive reactivity, which is contrary to the results of direct numerical solution of transient problem. Therefore, the reactivity determined at items 2-6 should be attributed to the error of the solution scheme.

4. Error of the Second Combined Approximation

Based on the results of the two tasks, it can be concluded that the solution of transient problem with the second combined approximation provides the most accurate results. Reactivity of this solution scheme contains an error because it is determined from the asymptotic state. In fact, reactivity is not constant in transients; it varies until a reactor achieves an asymptotics on delayed neutrons that can take a long time (hundreds of seconds).

Figure 3 shows graphs of power and reactivity during simulation of drop of all control rods to the reactor core obtained by direct numerical solution of transient neutron transport equation. For about 10 min. after ending of the control rod movement the maximum error of reactivity in the current time could reach 5.6% from its asymptotic value. In this case the established value of reactivity is identical to its estimate in steady state.

Similar calculations were conducted for experiments of insertion into the core of various combinations of control rods. Using results of processing the data, similar to those shown in Fig. 3, a graph of the maximum error of reactivity, defined by the second combination approximation from the magnitude of the perturbation of the reactor, was constituted. The results are given in Fig. 4.

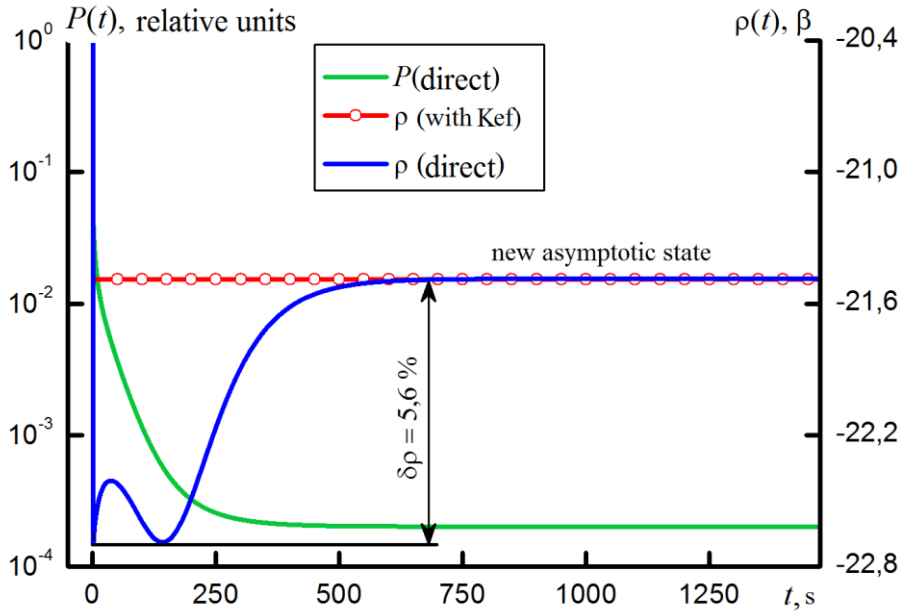

Fig. 3. Graphs of power P and ρ reactivity when drop of all control rods to the reactor core that were obtained by direct numerical solution of transient neutron transport equation and by the second combined approximation

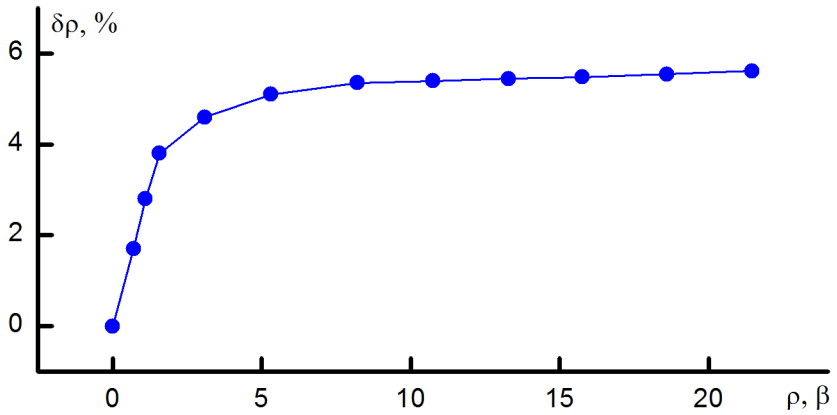

Fig. 4. Evaluation of the maximum error of the reactivity δρ, obtained from consideration of conditionally critical state from the magnitude of the perturbation ρ

As it can be seen from Fig. 4 that the error of reactivity is proportional to magnitude of the perturbation and reaches its maximum value of 5.6% in case of insertion into the reactor perturbations in -21,5 β_{ef}. This error is commensurate with the neutronic constant component of the error in defining of β_{ef}.

Conclusion

In this paper the analysis of the solution of transient neutron transport equation in different approximations is considered. Two tasks intended to reveal the accuracy of different solution

schemes were calculated. Two combined approximations based on the algorithm of the improved quasi-static approximation with different way of reactivity determination were presented and analyzed to improve the accuracy of the results.

In the paper the analysis of solution errors of each algorithm is presented. It is found that solutions obtained using the second combined approximation are preferred because of high solution accuracy and admissible time expenditure. Its errors in determining reactivity were investigated further.

References

[1] Asatryan, D. S., Belov A. A., Belousov V. I., Bereznev V. P., Ivchenko D. V., Seleznev, E. F., Chernova I. S., Kartashov, K. V., Peregudov A. A., Raskas K. F., Semenov M. Yu., Sorokin A. P., Tsibulya A. M., Yakunin, A. A., Drobyshev Yu. Yu., Karpov S. A., Fedorov I. V., Modules of Transient Calculation of Kinetics Problems in the GEFEST800 Software Package, Atomic energy, 2015, vol. 119, № 1, pp. 3-7.

[2] Panov, I. S., Seleznev, E. F., Modules of Transient Calculations of Kinetics in Complex GEFEST-800, XXV scientific-technical seminar. Neutron-physical problems of nuclear power (Satronica-2014), Obninsk, FSUE SSC RF IPPE. – 2014.

[3] Seleznev, E. F., Asatryan D. S., Belov A. A., Belousov V. I., Bereznev, V. P., Vasekin V. N., Need A. S., Panova I. S., Neutronics Codes, “Innovative designs and technologies of nuclear power”, International Scientific and Technical Conference: Collection of Papers, Moscow, JSC “NIKIET”, 2012.

[4] Seleznev, E. F., Kinetics of Fast Reactors, M., Science, 2013.

[5] Y. Shimazu, T. Takeda, W.F.G. van Rooijen, Development of a Three-Dimensional Kinetics Code for Commercial-Scale FBR Full Core Analysis, material international conference “PHYSOR 2014 - The Role of Reactor Physics toward a Sustainable Future”, Kyoto, Japan, September 28 - October 3, 2014.

[6] Bell D., Glasstone S., Theory of Nuclear Reactors, Moscow, Atomizdat, 1974.

[7] Stambul E. A., Application of Perturbation Theory in the Physics of Nuclear Reactors, Moscow, Atomizdat, 1976.

[8] Chernova, I. S., Modification of the Improved Quasi-Static Approximation for the Solution of Transient Neutron Transport Equation, proceedings of the XVII scientific school of young scientists of IBRAE ran, Moscow: IBRAE, 2016.