# Optimization problem for characteristics of fast reactors operating in a closed fuel cycle

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**Abstract.** Results of the analysis of the optimization problem for characteristics of fast reactors operating in a closed fuel cycle are presented. The optimization problem is proposed to solve in two stages. At the first stage, multiple calculations of characteristics of fast reactors are carried out using a variety of characteristics of the fuel returned to reactor from a closed nuclear fuel cycle, such as the content of fission fragments, the uncertainty in determination of the concentration of fissile nuclides in the returned fuel, etc. At the second stage, the optimization problem is solved directly by selecting the most appropriate characteristics of the reactor from multiple sets of obtained characteristics. As an example of solving the optimization problem the reactor EBR-II with a metal fuel returned to the reactor with the contents of the fission products after recycling is considered. The influence of the uncertainty in determination of the nuclides concentration on the reactor characteristics is shown.

Key words: characteristics of fast reactors, closed fuel cycle, optimization problem.

#### 1. Introduction

Results of the analysis of the optimization problem for characteristics of fast reactors operating in a closed fuel cycle are presented.

The optimization problem is proposed to solve in two stages. At the first stage, multiple calculations of characteristics of fast reactors are carried out using a variety of characteristics of the fuel returned to reactor from a closed nuclear fuel cycle, such as the content of fission fragments, the uncertainty in determination of the concentration of fissile nuclides in the returned fuel, etc. At the second stage, the optimization problem is solved directly by selecting the most appropriate characteristics of the reactor from multiple sets of obtained characteristics.

There are different approaches to solve the optimization problem for an objective function, which does not have predetermined analytical dependence on its parameters. One way or another, multiple calculations of function values at different sets of parameters are assumed. The following approach seems to be the least complicated in the case of a large number of optimization variables. Analytical approximation of the dependence is based on a previously calculated set of examples of the parameter values corresponding to the function values. Finally, when the approximation is constructed, the classic gradient methods are used to solve the optimization problem.

As an example of solving the optimization problem the reactor EBR-II with a metal fuel returned to the reactor with the contents of the fission products after recycling is

considered. The optimization problem has been examined earlier on the basis of the nuclide concentrations uncertainties in the fuel composition at a burnup and recycling in [2].

### 2. Initial data

The paper presents the results of the analysis of uncertainties for calculating the nuclide composition of the fast reactors fuel and uncertainties of definition of this composition in the external fuel cycle.

The main contributions to the uncertainties of nuclide composition are associated with:

- Uncertainties in the reactor operation and time of fuel cycle;
- Uncertainties in nuclides purification coefficients at the CNFC stages in the recycling of spent fuel.

Uncertainties in the reactor operation are caused by:

- Uncertainties in the cross sections of isotope kinetics and reactor calculations;
- Uncertainties in the decay data of nuclides.

Solution of the nuclide kinetics problem was performed using program BPSD [2], which provides nuclides concentration uncertainties simultaneously with their concentrations. Data directly from the EBR-II [1] and the data of paper [3] for the loss of nuclides in the recycling of fuel by pyroelectrochemical way were used to take into account the passage of fuel by CNFC stages and to estimate an error of nuclides concentrations in recycled fuel.

As an example of solving the optimization problem the reactor EBR-II with a metal fuel returned to the reactor with the contents of the fission products after recycling was chosen.

The EBR-II complex consisted of a nuclear power plant with a fast reactor and regeneration facilities of spent fuel and manufacture facilities of fuel assemblies. The reactor had a tank layout and has served as a bench for fuel and materials tests. The fuel was metallic uranium alloy with 5% (by weight) of fissium, which was obtained by pyrometallurgical reprocessing of fuel according to the technology specially developed for the EBR-II. A mass content of fissium is: Mo-2,4%; Ru-1,9%; Rh-0,3%; Pd-0,2%; Zr-0,1%; Nb-0,01%. Burnup over 11% h.a. was achieved at the reactor (two fuel assembly with fuel burnup was in the test model proposed by the IAEA - EBR-II - SHRT-45R). The reason for achieving those deep burnup in the metal fuel is related to its artificially created porosity. Fuel elements were produced with a large gap between the fuel and the shell, so that the "smudged" fuel density was about 75% of the theoretical one.

The EBR-II reactor with a metal fuel passing procedure for processing and returned to the reactor with the contents of the fission products [1] did not possess the optimal characteristics, since was exploratory. Реактор EBR-II с металлическим топливом, проходящим процедуру переработки и возвращаемым в реактор с содержанием части продуктов деления[1] не отличался оптимальными характеристиками, так как был исследовательским. Nevertheless, we will attempt to construct optimization problems on the example of the analysis of its neutronic characteristics and characteristics of the external fuel cycle during reprocessing its fuel.

## 3. Calculation results

The reactor test proposed by the IAEA (ANL) [1] allows us to analyze its parameters. Thus, Figure 1 shows the results of calculations of criticality, energy release and other characteristics of it in comparison with the calculations of other research groups.



FIG.1 Multiplication factor in EBR-II (model SHRT-45R) produced by different research groups (Argonne National Laboratory- ANL (USA), University of Fukui (Japan), Institute of Technology in Karlsruhe - KIT (Germany) code Euranos, Institute named Paul Scherer - PSI (Switzerland)) and us (the G1 - module of neutronic calculations in the diffusion multi-group approximation in three-dimensional hexagonal geometry) using various approximations (transport: ANL, KIT, PSI and diffusion: ANL, Fukui, G1) and various constants databases (ANL (ENDF-B), KIT (ENDF-B), PSI (ENDF-B) and G1 (ABBN-93)).

Figure 2 shows the energy release in the fuel assembly along the line A in the direction from the center of the reactor, obtained in the ANL, KIT (Euranos) and authors (G1).



FIG. 2 Energy release in the fuel assembly from the central fuel assembly in the direction A.

The breeding ratio of the core amounted to 0.050 and reactor breeding ratio in general amounted to 0.285. Follows from the data in Figures 1 and 2, we can conclude a good agreement between the calculated data obtained by G1 code and data from other calculations.

Table 1 provides the neutrons balance in the reactor. Table 2 provides the balance of neutron capture in the reactor over nuclides.

Reactor zone	Fission	Capture	Leakage
reactor	41.3	47.9	10.8
core	40.0	12.3	-
blanket	1.3	12.7	-
cells out of core and blanket	-	22.9	-
(steel + sodium)			

TABLE 1: NEUTRONS BALANCE IN THE REACTOR, %

TABLE 2: FRACTION OF NEUTRONS CAPTURES IN THE REACTOR OVER	THE
NUCLIDES	

Nuclide	Capture, %						
U234	1.039E-03	Am241	7.171E-05	La139	1.943E-03	Fe	9.7791
U235	46.729	Am242m	6.936E-09	Nd148	1.822E-03	Mn	3.1820
U236	0.12215	Am243	5.375E-10	Zr	1.923E-03	Cr	3.9691
U238	15.903	Cm242	3.979E-09	Mo	0.29236	Ni	3.4313
Np237	4.123E-03	Cm243	1.439E-11	Ru	0.34656	С	2.615E-04
Pu236	6.950E-09	Cm244	1.773E-12	Rh	0.10021	Si	2.210E-03
Pu238	5.342E-05	Cm245	1.386E-14	Pd	3.721E-02	Р	2.111E-03
Pu239	1.1772	Cm246	1.878E-19	Nb	1.213E-03	S	6.578E-03
Pu240	8.654E-03	FP235	0.30234			Cu	0.81375
Pu241	9.458E-05	FP239	7.629E-03	B-10	2.40171	Na	0.34750
Pu242	1.274E-07			B-11	4.733E-05		

From the Table 2 it follows that the fraction of captures on fissium nuclides does not exceed even 1%. The fraction of captures on actinides, including Pu239, does not exceed even 1.2%.

Figure 3 shows the dependence of the breeding ratio of the core on the fuel burnup in fuel assembly of one of the core series. Since the initial value of the parameter is small due to the high fuel enrichment, so the drop reaches a value of 10% with an increment of burnup on this same fraction.



FIG. 3 Dependence of the breeding ratio of the core (KBA) on the burning of fuel in fuel assembly of the 4th series of EBR-II

Figure 4 shows the dependence (dk/k / dro/ro) for fuel nuclides. The same parameter for the fission products and nuclides of fissium is presented in Figure 5.



FIG. 4 Dependence (dk/k / dro/ro) for actinides from the fuel of the core



FIG. 5 Dependence (dk/k / dro/ro) for non-fissile nuclides in the fuel of the core (without nuclides of steel)



FIG. 6 Dependence dkva/dro for nuclides in the fuel of the core



FIG. 7 Dependence dkva / dro with a negative sign for some nuclides in the fuel of the core

From Figure 7 it follows that we must first reduce the presence of U235, that is, to reduce the initial fuel enrichment to improve the KBA.



FIG. 8 Dependence dKB/dro for nuclides in the fuel of the core

A positive dependence of changes in sensitivity of the breeding ratio on nuclides concentrations is typical basically for even nuclides, which is associated with the "extrusion" of the field of neutrons from the reactor core fuel assembly in the side of the screen and the main accumulation of secondary fuel in these assemblies. This is not observed in power reactors due to the main accumulation of secondary fuel in the fuel assembly of the core.



FIG. 9 Dependence dKV / dro with a negative sign for the nuclides in the fuel of the core

The constructed sensitivity coefficients allow to easily solve simple tasks with a small number of parameters. For example, according to Figure 3, the smaller burnup in the fuel, the higher its KBA. It provide increasing of reactor criticality by decreasing in the proportion of Ru in fissium (Fig. 5), for example, by replacing it with Zr, etc. Table 3 presents data on the uncertainties of nuclides concentration at fuel burnup obtained as in [2].

Nuclide	δρ	Nuclide	δρ	Nuclide	δρ
U234	0.48	Pu241	0.73	Cm246	10.60
U235	0.5e-02	Pu242	1.64	La139	0.3e-02
U236	0.33	Am241	0.87	Nd148	1.76
U238	0.3e-03	Am242m	3.73	Zr	1.25
Np237	1.06	Am243	3.51	Mo	3.55
Pu236	7.91	Cm242	1.71	Ru	0.05
Pu238	1.54	Cm243	2.43	Rh	0.49
Pu239	0.61	Cm244	5.32	Pd	1.61
Pu240	0.70	Cm245	7.72	Nb	0.15

TABLE 3: UNCERTAINTIES OF NUCLIDE CONCENTRATIONS AT BURNUP,  $\delta \rho$ , %

The maximum uncertainty in the concentrations is observed in nuclides with a minimum concentration (Cm246). Uncertainty of determining Pu239 concentrations is 0.61% and also should be considered as significant for the fissile nuclide. Minimum uncertainty occurs in the main fissile nuclide (U235), which is associated with a very high enrichment of fresh fuel - 67% [1].

Table 4 presents an analysis of the composition of fissium in the reactor and directly into the fission products at a burnup. Appears from the table that zirconium was extracted a great extent from the fissium during recycling of irradiated fuel. Whereas it improved criticality conditions in the reactor by increasing its proportion, for example, by Mo or Ru. But it is possible that the reason for the extraction of zirconium is related to the peculiarities of behavior of the U + fissium alloy under irradiation.

Nuclide	Proportion in	Proportion in fragments	Proportion in
	fissium,%	of nuclides,%	fissium/Proportion in
			fragments
Zr	2.160	44.46	0.05
Mo	50.530	28.40	1.78
Ru	38.217	20.07	1.90
Rh	5.362	2.53	2.11
Pd	3.519	2.57	1.37
Nb	0.212	1.96	0.11

## TABLE 4: PROPORTION OF FISSIUMNUCLIDES IN THE "FRESH" FUEL AND FISSION FRAGMENTS, % NUCLEAR CONCENTRATIONS

Data of paper [3] were used as the losses of nuclides data in the processing. The loss estimates were used for processing by pyroelectrochemical method. Loss of fuel isotopes and fission products in this case reach values of 0.3% for U, 0,7% for Pu. Proportion of actinides and fission products in a recycled fuel reaches 50% for Np; 20% for Am; 5% to Cm; as for the Zr, Ru, Rh, Pd, Nb, Tc; only 0.2% for Mo [3].

Impact of the concentration uncertainty of its constituent elements on the characteristics of the reactor is not significantly because of the low content of fissium in EBR-II fuel.

Impact of the concentration uncertainty of Pu and its loss on the recycling characteristics of the reactor is minimized by its non-return to the reactor as a part of returned "fresh" fuel. According to [3] it is necessary to consider changes in the technology of the "fresh" fuel made after processing in addition to the influence of external conditions on the reactor fuel cycle characteristics when considering a bunch reactor - external fuel cycle. For example, to take into account such parameters as the level of gamma radiation from the fuel assembly with the specified fuel, etc.

Thus, the formulation of the optimization problem for the reactor parameters and CNFC can be formulated as follows:

Reactor parameters: Keff; reactivity margin; KBA; KB; reactivity ratios.

CNFC parameters: coefficients of nuclides losses; coefficients of nuclides purification.

Technological parameters of the "fresh" and spent fuel: activity of fuel assembly; radiation level of fuel assembly.

The task is to minimize the impact of CNFC characteristics on criticality, loss of reactivity during burnup and loss of reproduction.

This problem can be extended to an assessment of the influence of CNFC passage of emergency situations on reactor reactivity through the requirement for a level not lower than required by the analysis of emergencies.

Notice that in the general case an impact of uncertainties becomes significant with a decreasing in the quality of purification and increasing the proportion of secondary fuels. In this case, the problem of optimization of the reactor, in view of its comprehensiveness, becomes a difficult mathematical problem, which requires the appropriate mathematical apparatus for its solution. Optimization problem can be formulated as the problem of finding the argument of the function F(x) in which the function takes the optimum value. The function F(x) in this case is called the objective function. It is possible to apply the method of gradient descent [4] if an analytical expression of F (x) is known, and the function is differentiable respect to the argument x. Method of gradient descent is an iterative procedure, which allows us to find the value of the argument at a local extremum. Its result depends on

the choice of the initial value as well as the choice of the parameters used by an iterative procedure. Method of gradient descent can be used in many applied problems in which there is no principal necessity to find the best solution, but to find a good enough.

It is more difficult in the case when an analytical expression of the objective function is unknown, but its value may be obtained at any given point, for example, by numerical calculation. So the solution can be reduced to method of gradient descent, for example, if there are physical assumptions allow to build an approximation of the objective function within a physical model.

If there is no such assumptions, so in the most general case, there are two possible solutions to the problem: firstly, to evaluate function at the point obtained by the analysis of its values in the previous points [5]. Such methods tends to become significantly expensive from a computational point of view in the case of a large number of optimization parameters (a few dozen).

The second option, which is scheduled to be used by authors in the future work, is associated with optimization of characteristics of fast power reactor and CNFC parameters as follows. At first, the value of the objective function are considered in the region of interest change of optimization settings, then this set of parameters is carried out numerical approximation [6]. This results in an analytical approximation of the objective function. It permits to reduce the problem to the classical gradient descent method

## 4. Conclusion

Analysis of the EBR-II reactor characteristics indicates that as this is a research reactor, the optimization problem is very difficult to set. The reactor is poor suitable for optimization because the influence of returned material to the reactor (fissium) is insignificant due to high fuel enrichment. Change of the reactivity coefficients at burnup is insignificant and does not exceed 3%, i.e. it actually lies within measurement error.

An optimization problem leads to a simple solution - the requirement of reducing fuel enrichment while preserving the operational parameters of the reactor.

The problem can be set differently for research reactors. Namely, an increase in the "quality" of the reactor - relation of the neutron flux density to the power unit. If the capacity of the existing reactor is difficult to change because of the organization adopted in the project of heat removal, so the rise of the neutron flux density is possible at the same incidence of enrichment, but it will increase neutron leakage and loss of reactivity greatly increase the speed to the detriment of the campaign duration.

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